Transport simulation of the linear sigma model*

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The most popular tool for *DCC* studies has been the linear σ model which describes the chiral field $\phi = (\sigma, \pi)$ by means of a simple effective quartic interaction. This framework has provided instructive insight into the feaures of the non-equilibrium *DCC* dynamics. However, the field framework has several disadvantages relative to a particle formulation: 1) the interpretation of the state is less obvious, 2) the coupling to the hadronic environment is harder to include, and 3) it is impractical to treat the global expansion that is expected to occur in the situations of interest (a local boost endows the field with strong undulations which can only be removed by special coordinate transformations in a few idealized expansion scenarios). We therefore seek to recast the field formulation of the linear σ model into a particle description, thereby paving the way for embedding the chiral dynamics into microscopic transport codes so that more realistic simulations can be carried out.

The first step in this program is to apply a mean-field approximation to the full field equation [1]. The field is thereby separated into two parts, $\phi = \underline{\phi} + \delta\phi$ where the smooth part, $\underline{\phi}$, is identified with the local *order parameter* and the fluctuations $\delta\phi$ represent *quasi-particle* excitations relative to that local vacuum. This yields Klein-Gordon dispersion relations containing an effective mass tensor depending on both agitation and order parameter. The second step is to consider the Wigner transform of the quasi-particle part of the field which is governed by a Vlasov equation. Finally, invoking a semi-classical approximation, we represent the associated quasi-particle phase-space distribution by a swarm of test particles. This yields equations of motion for the phase-space trajectory of each test particle, in addition to the equation governing the rotation of its $O(4)$ chiral orientation.
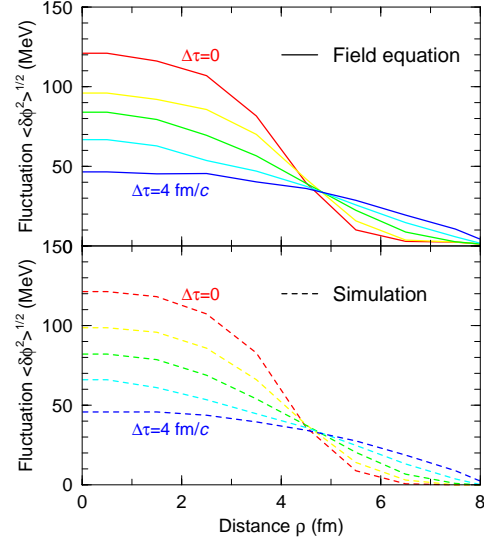
Figure 1: The quality of the treatment is illustrated for an initially hot cylinder subjected to longitudinal Bjorken expansion. The figure shows the evolving profile of the *field fluctuation* for the expanding cylinder, after a certain proper time has elapsed, $\tau = 0, 1, 2, 3, 4$ fm/c, as calculated either with the full field equation (solid) or the quasi-particle transport simulation (dashed).

The above transport treatment is directly applicable to scenarios where the quasi-particle trajectories remain within the classically allowed region of phase space. In such a case, all quasi-particle energies remain positive and it is straightforward to solve the equations of motion for the quisi-particle trajectories, in conjunction with the field equation for the order parameter. However, but special considerations are needed whenever a quasi-particle mode grows unstable.

The reformulation of the linear σ model in terms of medium-modified quasi-particles will make it practically easier to simulate *DCC* dynamics in the complicated dynamical environments of high-energy collisions,

[1] J. Randrup, Nucl. Phys. A616 (1997) 531